Vocabulary
Write the term that best completes each statement.

1. The terminal ray of an angle in standard position is the ray with its endpoint at the origin that is not the _____________.

2. The _____________ of a periodic function is one half the absolute value of the difference between the maximum and minimum values of the function.

3. An angle is in ______________ when the vertex is at the origin and one ray of the angle is on the x-axis.

4. A ________________ is a function whose values repeat over regular intervals.

5. The ________________ of a periodic function is a reference line whose equation is the average of the minimum and maximum values of the function.

6. The ________________ of a periodic function is the length of the smallest interval over which the function repeats.

7. The measure of an angle in standard position is the amount of rotation from the initial ray to the ______________.
Problem Set

Sketch a graph for the function that is described in each exercise.

1. On an obstacle course, a person must jump from a platform onto a giant paddle wheel that has a diameter of 17 yards. The person must then ride the paddle wheel for at least one revolution before jumping off the wheel and back onto the platform. Diana rides the wheel for three revolutions before jumping off. The platform is nine yards off the ground. Sketch a graph of the model of the height of Diana above the ground with respect to the number of revolutions.

2. A frog clings to the edge of a paddle of a wheel that is spinning behind a paddle boat. The wheel has a diameter of 16 feet. The frog hops on the edge of the wheel just after it comes out of the water and manages to stay on for four revolutions before falling into the water. Sketch a graph of the model of the height of the frog above water with respect to the number of revolutions.
3. A water park has just opened the first ever underwater Ferris wheel. It is 15 feet in diameter. Each rider gets into a completely waterproof car at ground level and rides around for an underwater adventure for five revolutions. Then, they exit the ride at ground level. Sketch a graph of the height of the rider in relationship to ground level with respect to the number of revolutions.

4. A fly leaves its spot on the top of a bookshelf and lands a horizontal distance of 1.5 feet away on the end of a blade of a ceiling fan. It rides around for four revolutions before flying back to its original spot on the bookshelf. The ceiling fan has a diameter of four feet. Sketch a graph of the model of the distance the fly is from the spot on the bookshelf with respect to the number of revolutions.
5. Franco’s favorite ride at the fair is the 32-foot-diameter carousel. He hops on the black horse while his grandmother stands outside the gate at the nearest point to him. Before the ride begins, Franco is 10 feet from his grandmother. The carousel goes around 8 times during the ride and then stops in its original position. Sketch a graph of the model of the distance Franco is from his grandmother with respect to the number of revolutions.

6. A bug hops on the tip of a 7-inch-long second hand of a mantel clock while it is pointing to the 12 and rides the second hand around the clock. The bug stays on the second hand for 5 full minutes before jumping off. Sketch a graph of the model of the height of the bug above the mantel with respect to the number of revolutions. Assume that the distance between the mantel and the 6 on the clock is 4 inches.
Determine whether each graph represents a periodic function over the interval shown. If so, identify the period $p$.

7. Yes. The graph represents a periodic function with period $p = 2$.

9. 

10. 

Determine the midline and amplitude of each graph.

13. The midline is \( y = 0 \) because \( \frac{3 + (-3)}{2} = 0 \).

The amplitude is 3 because \( \frac{1}{2} | 3 - (-3) | = 3 \).
Name _____________________________ Date __________

15. 

16. 

17. 

18.
19.

20.
Two Pi Radii
Radian Measure

Vocabulary
Complete each sentence with the word(s) that makes it true.

1. A unit circle has a radius of \( \pi \) units.

2. A symbol used to identify the central angle measure of a circle in standard position is \( \theta \).

3. There are \( 2\pi \) \( \frac{\pi}{2} \) in 360°.

Problem Set
Calculate the arc length and the radian measure of each angle in a circle with the given dimensions.

1. \( \theta = 30^\circ \); radius = 3 units
   
   The radian measure is \( \frac{\pi}{6} \) radian.

   \[
   \text{Arc length} = 2\pi (3) \cdot \frac{30^\circ}{360^\circ} = (6\pi) \cdot \frac{1}{12} = \frac{\pi}{2} \text{ units}
   \]

   \[
   \text{Radian measure} = \frac{\pi}{2} \cdot \frac{3}{3} = \frac{\pi}{2} \div 3 = \frac{\pi}{6}
   \]
3. $\theta = 45^\circ$; radius = 5 units

4. $\theta = 100^\circ$; radius = 6 units

5. $\theta = 135^\circ$; radius = 4 units

6. $\theta = 180^\circ$; diameter = 10 units
7. $\theta = 90^\circ$; diameter = 12 units  
8. $\theta = 15^\circ$; diameter = 15 units  
9. $\theta = 60^\circ$; diameter = 8 units  
10. $\theta = 300^\circ$; diameter = 3 units
Estimate the degree measure of each central angle given in radians in a unit circle. Explain your reasoning.

11. 4 radians
   The degree measure of 4 radians is a little less than 240°.
   
   Four radians is two-thirds of six radians, which is a little less than 360°. Two-thirds of 360° is 240°. So, four radians is a little less than 240°.

12. 2 radians

13. 5 radians

14. 6 radians

15. 1 radian
16. 2.5 radians

17. 3.25 radians

18. 5.1 radians

19. 1.57 radians

20. 6.28 radians
Convert each radian measure to degrees. Round each answer to the nearest hundredth.

21. 5 radians

Degree measure = 5 radians \cdot \frac{180°}{\pi \text{ radians}}

= \frac{900°}{\pi}

≈ 286.48°

22. 3 radians

23. 6 radians
24. 2 radians

25. 4 radians
26. 1.9 radians

27. 5.8 radians
28. 2.3 radians

29. 4.75 radians
30. 3.4 radians
Convert each degree measure to radians. Write each answer as a simplified ratio in terms of $\pi$.

31. $121^\circ$
   
   Radian measure $= 121^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$
   
   $= \frac{121\pi}{180}$ radians

32. $85^\circ$

33. $204^\circ$

34. $66^\circ$

35. $18^\circ$

36. $196^\circ$
37. 320°
38. 256°
39. 102°
40. 305°
Triangulation
The Sine and Cosine Functions

Vocabulary
Write a definition for each term in your own words.

1. sine function

2. cosine function

3. trigonometric function

4. periodicity identity
Problem Set

Use the unit circle to determine each value.

1. \( \sin\left(\frac{\pi}{2}\right) \)

2. \( \sin\left(\frac{5\pi}{6}\right) \)

\[ \sin\left(\frac{\pi}{2}\right) = 1 \]

3. \( \sin(2\pi) \)

4. \( \sin\left(\frac{5\pi}{4}\right) \)

5. \( \cos\left(\frac{\pi}{4}\right) \)

6. \( \cos\left(\frac{2\pi}{3}\right) \)

7. \( \cos\left(\frac{3\pi}{2}\right) \)

8. \( \cos\left(\frac{7\pi}{6}\right) \)
The measure of an angle is given in radians. Determine the coordinates of the point at which the terminal ray of each angle intersects the unit circle.

9. \( \pi \) radians

10. \( \frac{\pi}{3} \) radians

(\(-1, 0\))

11. \( \frac{3\pi}{4} \) radians

12. \( \frac{\pi}{6} \) radian

13. \( \frac{4\pi}{3} \) radians

14. \( \frac{7\pi}{4} \) radians

15. \( \frac{5\pi}{3} \) radians

16. \( 2\pi \) radians

17. 0 radians

18. \( \frac{11\pi}{6} \) radians
Evaluate the sine and cosine of the supplement of the given measure. Explain your reasoning.

19. \( \theta = \frac{5\pi}{6} \)
   
   \[
   \sin\left(\frac{\pi}{6} \text{ radian}\right) = \frac{1}{2} \\
   \cos\left(\frac{\pi}{6} \text{ radian}\right) = \frac{\sqrt{3}}{2} 
   
   \]

   The supplement of \( \frac{5\pi}{6} \) radians or 150° is 180° − 150° or 30°. This is equivalent to \( \frac{\pi}{6} \) radian.

20. \( \theta = \frac{\pi}{4} \)

21. \( \theta = \frac{2\pi}{3} \)

22. \( \theta = \frac{3\pi}{4} \)

23. \( \theta = \frac{\pi}{2} \)

24. \( \theta = \frac{\pi}{6} \)
Pump Up the Amplitude
Transformations of Sine and Cosine Functions

Vocabulary
Write the word(s) that best completes each statement.

1. The ________________ of a periodic function is the reciprocal of the period and specifies the number of repetitions of the graph of a periodic function per unit.

2. For periodic functions, a horizontal translation is called a ________________.

Problem Set
Determine the amplitude of each graph.

1. \( y = 3 \sin(x) \)

2. \( y = \frac{1}{2} \cos(x) \)

amplitude = 3
3. \( y = \cos\left(\frac{1}{4}x\right) \)

4. \( y = \sin(3x) \)

5. \( y = -0.5 \cos(x) \)

6. \( y = -4 \sin(x) \)
Determine the period and frequency of each graph.

7. \( y = \frac{3}{4} \cos(x) \)

The period is \( \frac{2\pi}{1} \), or \( 2\pi \) radians.

The frequency is \( \frac{1}{2\pi} \).

8. \( y = 2 \sin(x) \)

9. \( y = \sin\left(\frac{1}{3}x\right) \)

10. \( y = \cos(3x) \)
11. \( y = \sin(4x) + 1 \)

12. \( y = -2 \cos\left(\frac{2}{3}x\right) - 3 \)

Describe the transformation performed on the graph of the basic function \( f(x) \) to produce the graph of \( g(x) \).

13. \( f(x) = \sin(x), \quad g(x) = 6 \sin(x) \)

   The graph of \( f(x) \) is stretched vertically by a factor of 6.

14. \( f(x) = \sin(x), \quad g(x) = \sin\left(\frac{2}{5}x\right) \)

15. \( f(x) = \sin(x), \quad g(x) = \sin(x) - 2 \)

16. \( f(x) = \sin(x), \quad g(x) = \sin(x + \pi) \)
Name ___________________________________________________________ Date ____________

17. \( f(x) = \cos(x) \), \( g(x) = \cos(5x) \)

18. \( f(x) = \cos(x) \), \( g(x) = \frac{1}{6} \cos(x) \)

19. \( f(x) = \cos(x) \), \( g(x) = \cos\left(x - \frac{\pi}{2}\right) \)

20. \( f(x) = \cos(x) \), \( g(x) = \cos(x) + 7 \)

Use what you know about transformations to sketch the graph of each given function.

21. \( y = \frac{1}{2} \cos(x) + 1 \)

22. \( y = -\cos(2x) \)
23. \( y = \frac{1}{4} \sin\left(x - \frac{\pi}{2}\right) - \frac{1}{2} \)

24. \( y = \frac{3}{2} \sin(3x) + 3 \)

25. \( y = \frac{3}{4} \sin(2x + \pi) + 2 \)

26. \( y = 2 \cos\left(\frac{1}{2}x - \frac{\pi}{2}\right) - 1 \)
Farmer’s Tan
The Tangent Function

Vocabulary

1. Provide an example of a tangent function.

2. Explain how the tangent function is related to the sine and cosine functions.

Problem Set

Calculate the tangent of each angle given the cosine and sine of the angle.

1. \( \sin(\theta) = \frac{3}{5}, \cos(\theta) = \frac{4}{5} \)

   \[
   \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{3}{5} \div \frac{4}{5} = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}
   \]

2. \( \sin(\theta) = \frac{7}{25}, \cos(\theta) = \frac{24}{25} \)

3. \( \sin(\theta) = \frac{8}{17}, \cos(\theta) = \frac{15}{17} \)

4. \( \sin(\theta) = \frac{12}{13}, \cos(\theta) = \frac{5}{13} \)
5. \( \sin(\theta) = \frac{40}{41} \cos(\theta) = \frac{9}{41} \)

6. \( \sin(\theta) = \frac{20}{29} \cos(\theta) = \frac{21}{29} \)

7. \( \sin(\theta) = \frac{2\sqrt{5}}{5}, \cos(\theta) = \frac{\sqrt{5}}{5} \)

8. \( \sin(\theta) = \frac{2\sqrt{13}}{13}, \cos(\theta) = \frac{3\sqrt{13}}{13} \)

Evaluate each tangent function by using the relationship between the tangent function and the sine and cosine functions.

9. \( \tan\left(\frac{\pi}{2}\right) \)

\( \tan\left(\frac{\pi}{2}\right) = \frac{1}{0} \)

= undefined

10. \( \tan(\pi) \)

11. \( \tan\left(\frac{5\pi}{4}\right) \)

12. \( \tan\left(\frac{2\pi}{3}\right) \)
Lesson 13.5 Skills Practice

Name ___________________________ Date __________

13. \( \tan(0) \)  

14. \( \tan\left(\frac{3\pi}{2}\right) \)

15. \( \tan\left(\frac{\pi}{3}\right) \)  

16. \( \tan\left(\frac{4\pi}{3}\right) \)

17. \( \tan\left(\frac{7\pi}{6}\right) \)  

18. \( \tan\left(\frac{\pi}{6}\right) \)

19. \( \tan\left(\frac{3\pi}{4}\right) \)  

20. \( \tan\left(\frac{5\pi}{6}\right) \)
Compare the graph of each transformation to the graph of \( \tan(x) \) shown below. Then, answer the question and explain how you determined your answer.

21. Does the graph below represent the function \( y = \tan(3x) \) or \( y = 3 \tan(x) \)?

The graph represents the function \( y = 3 \tan(x) \), because multiplying the function by 3 causes a vertical stretch.
22. Does the graph below represent the function $y = \tan(3x)$ or $y = \tan \left( \frac{1}{3}x \right)$?

23. Does the graph below represent the function $y = -2 \tan(x)$ or $y = -\tan(x)$?
24. Does the graph below represent the function $y = \tan\left(\frac{x - \frac{1}{2}}{2}\right)$ or $y = \tan\left(\frac{x - \pi}{2}\right)$?

25. Does the graph below represent the function $y = \tan(x) - 1$ or $y = \tan(x) + 1$?
26. Does the graph below represent the function \( y = 2 \tan(x) - 1 \) or \( y = \frac{1}{2} \tan(x) - 1 \)?

Graph each transformation of the function \( \tan(x) \).

27. \( y = \tan(2x) \)

28. \( y = -\frac{1}{2} \tan\left(\frac{1}{3}x\right) \)
29. \( y = \tan\left(x + \frac{3\pi}{2}\right) \)

30. \( y = \frac{1}{4} \tan(x) \)

31. \( y = \tan(x) - 2 \)

32. \( y = 2 \tan(x) + 3 \)