LESSON 10.1 Skills Practice

Name ________________________________________ Date __________

Small Investment, Big Reward
Exponential Functions

Vocabulary
Define each term in your own words.

1. exponential function

2. half-life

Problem Set
Write the explicit formula for each geometric sequence. Then, use the equation to determine the 10th term. Round answers to the nearest thousandth, if necessary.

1.  
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
<td>45</td>
<td>135</td>
<td>405</td>
<td>1,215</td>
<td>98,415</td>
</tr>
</tbody>
</table>

   \[ a_n = 5 \cdot 3^{n-1} \]
   \[ a_{10} = 5 \cdot 3^{10-1} \]
   \[ = 5 \cdot 3^9 \]
   \[ = 98,415 \]

2.  
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>100</td>
<td>50</td>
<td>25</td>
<td>12.5</td>
<td>6.25</td>
<td></td>
</tr>
</tbody>
</table>
### Lesson 10.1 Skills Practice

**3.**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1.25</td>
<td>1.563</td>
<td>1.953</td>
<td>2.441</td>
<td>3.052</td>
<td></td>
</tr>
</tbody>
</table>

**4.**

<table>
<thead>
<tr>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>0.8</td>
<td>0.64</td>
<td>0.512</td>
<td>0.410</td>
<td>0.328</td>
<td></td>
</tr>
</tbody>
</table>

**5.**

<table>
<thead>
<tr>
<th></th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.4</td>
<td>0.8</td>
<td>1.6</td>
<td>3.2</td>
<td>6.4</td>
<td>12.8</td>
<td></td>
</tr>
</tbody>
</table>

**6.**

<table>
<thead>
<tr>
<th></th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>27</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>1/3</td>
<td>1/9</td>
<td></td>
</tr>
</tbody>
</table>
Write an exponential function in general form to represent each geometric sequence. Evaluate the function for the given value of $n$. Round to the nearest thousandth, if necessary.

7. $a_n = 4 \cdot 2.5^{n-1}$  
   $n = 10$  
   \[ f(n) = 4 \cdot 2.5^{10} \]  
   \[ = 4 \cdot 2.5 \cdot \left(\frac{5}{2}\right)^{10} \]  
   \[ = 4 \cdot 2.5 \cdot \frac{2}{5} \]  
   \[ = 1.6 \cdot 2.5^n \]  
   \[ f(10) = 1.6 \cdot 2.5^{10} \]  
   \[ \approx 15,258.789 \]

8. $a_n = 0.3 \cdot 8^{n-1}$  
   $n = 3$  
   \[ f(n) = 0.3 \cdot 8^n \]  
   \[ = 0.3 \cdot 2.5 \cdot 5 \]  
   \[ = 1.6 \cdot 2.5^n \]  
   \[ f(10) = 1.6 \cdot 2.5^{10} \]  
   \[ \approx 15,258.789 \]

9. $a_n = 150 \cdot 0.8^{n-1}$  
   $n = 2$  
   \[ f(n) = 150 \cdot 0.8^n \]  
   \[ = 150 \cdot 0.8 \cdot \frac{1}{2} \]  
   \[ = 60 \cdot 0.8^n \]  
   \[ f(2) = 60 \cdot 0.8^2 \]  
   \[ \approx 46.08 \]

10. $a_n = 0.5 \cdot 1.25^{n-1}$  
    $n = 24$  
    \[ f(n) = 0.5 \cdot 1.25^n \]  
    \[ = 0.5 \cdot 1.25 \cdot \frac{5}{4} \]  
    \[ = 1.25 \cdot 0.5^{n-1} \]  
    \[ f(24) = 1.25 \cdot 0.5^{24} \]  
    \[ \approx 3.2 \]
11. \( a_n = 10 \cdot 4^{n-1} \)
   \( n = 7 \)

12. \( a_n = 100 \cdot 0.5^{n-1} \)
   \( n = 5 \)

Write an exponential function \( A(t) \), where \( t \) represents elapsed time, to represent each half-life situation. Then, use the function to complete each table. Round as necessary.

13. Elapsed Time (hours)    | 0 | 2 | 4 | 6 | 8 | 20  
Drug in Bloodstream (mg)   | 120 | 60 | 30 | 15 | 7.5 | 0.1172  
Number of Half-Life Cycles | 0 | 1 | 2 | 3 | 4 | 10  

\[
A(t) = 120 \left( \frac{1}{2} \right)^{\frac{t}{2}} \\
A(20) = 120 \left( \frac{1}{2} \right)^{\frac{20}{2}} \\
= 120 \left( \frac{1}{2} \right)^{10} \\
\approx 0.1172
\]

14. Elapsed Time (minutes)  | 0 | 5 | 10 | 15 | 20 | 100  
Bacteria Subject to Growth Inhibitor | 800 | 400 | 200 | 100 | 50 |  
Number of Half-Life Cycles | 0 | 1 | 2 | 3 | 4 |
15. Elapsed Time (years)  | 0  | 14  | 21  | 28  | 42  | 56  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Strontium in Rock Sample (grams)</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Number of Half-Life Cycles</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

16. Elapsed Time (years)  | 0  | 5,700 | 11,400 | 15,675 | 17,100 | 22,800 
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C-14 in Rock Sample (grams)</td>
<td>1</td>
<td>0.5</td>
<td>0.25</td>
<td>0.125</td>
<td>0.0625</td>
<td></td>
</tr>
<tr>
<td>Number of Half-Life Cycles</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
### 17.

<table>
<thead>
<tr>
<th>Elapsed Time (Days)</th>
<th>0</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>42</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rat Population Exposed to Virus</td>
<td>5000</td>
<td>2500</td>
<td>1250</td>
<td>625</td>
<td>313</td>
<td></td>
</tr>
<tr>
<td>Number of Half-Life Cycles</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

### 18.

<table>
<thead>
<tr>
<th>Elapsed Time (Hours)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participants in Tennis Tournament</td>
<td>256</td>
<td>128</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Number of Half-Life Cycles</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
We Have Liftoff!
Properties of Exponential Graphs

Vocabulary
Explain how the natural base $e$ is similar to and different from $\pi$.

Problem Set
Determine whether the given function represents exponential growth or decay. Explain your reasoning.

1. $f(x) = 8^x$
   - The function represents exponential growth because the base is greater than 1.

2. $f(x) = 0.2^x$

3. $f(x) = \left(\frac{5}{2}\right)^x$

4. $f(x) = 25^x$

5. $f(x) = \left(\frac{1}{6}\right)^x$

6. $f(x) = 7.5^x$
Complete each table and graph the exponential function.

7. \( f(x) = 2^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

8. \( f(x) = \left(\frac{1}{2}\right)^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Name ___________________________ Date ____________

9. \( f(x) = 1.1^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

10. \( f(x) = \left(\frac{5}{4}\right)^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
11. \( f(x) = 6^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

12. \( f(x) = 0.3^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Write an exponential function with the given characteristics.

13. increasing over \((-\infty, \infty)\)
   reference point \((-1, \frac{1}{9})\)
   
   Answers will vary.
   \(f(x) = 9^x\)

14. decreasing over \((-\infty, \infty)\)
   reference point \((1, \frac{2}{3})\)

15. end behavior: \(\text{as } x \to -\infty, f(x) \to 0\)
    as \(x \to \infty, f(x) \to \infty\)
   reference point \((2, 2.25)\)

16. decreasing over \((-\infty, \infty)\)
   reference point \((-2, 16)\)

17. increasing over \((-\infty, \infty)\)
   reference point \((-3, \frac{1}{8})\)

18. end behavior: \(\text{as } x \to -\infty, f(x) \to \infty\)
    as \(x \to \infty, f(x) \to 0\)
   reference point \((-4, \frac{81}{16})\)
Use the formula for compound interest to determine the amount of money in each account after interest is accrued.

19. An investor deposits $1,000 in an account that promises an annual interest rate of 5%, compounded at the end of each year. How much will be in the account after seven years?

There will be $1,407.10 in the account after seven years.

\[ A(t) = P\left(1 + \frac{r}{n}\right)^{nt} \]

\[ A(7) = 1,000\left(1 + \frac{0.05}{1}\right)^{7} \]

\[ = 1,000(1.05)^{7} \]

\[ \approx 1,407.10 \]

20. At the start of the school year, Fairview High School deposits PTA dues in an account that offers an annual interest rate of 3.5%, compounded at the end of each year. If $2500 is collected in PTA dues, how much money will the school have at the start of the next school year?

21. Kyle put $300 of his birthday money in the bank. The bank offers an annual interest rate of 4%, compounded twice a year. How much money will Kyle have after three years?
22. An investing group has $50,000 to invest. They put the money in an account that has an annual interest rate of 6%, compounded monthly. How much money will the group have at the end of 10 years?

23. Interest is compounded quarterly at Money Bank at an annual rate of 5.5%. A new client opens an account with $7200. How much money will be in the account at the end of six years?

24. Sasha wants to earn the maximum interest on her money. She decides to deposit $50 in two different banks for 90 days (3 months) to compare them before she deposits all of her money. She finds a bank that compounds interest daily at an annual rate of 2.2% and another bank that compounds interest monthly at an annual rate of 4.8%. With which bank will she earn more money?
Use the formula for population growth to predict the population of each city.

25. The population of Austin, Texas is growing at a rate of 3.9% per year. If the population in 2010 was approximately 790,000, what is the predicted population for 2015?

The population of Austin, Texas will be about 960,096 in 2015.

\[ N(t) = N_0e^{rt} \]

\[ N(5) = 790,000e^{0.039 \times 5} \]

\[ = 790,000e^{0.195} \]

\[ \approx 960,096 \]

26. The population of Boston, Massachusetts is growing at a rate of 1.8% annually. The population in 2013 was approximately 636,500. What is the predicted population for 2025?

27. The population of Charlotte, North Carolina in 2013 was approximately 775,000. If the annual rate of growth is about 3.2%, what is an approximation of Charlotte’s population in 2000?
28. The population of Beijing, China in 2012 was approximately 20,690,000 and is growing at an annual rate of about 5.5%. What is an approximation of Beijing’s population in 1980?

29. The population of Detroit, Michigan is decreasing at an annual rate of about 0.75%. Detroit’s population in 2013 was approximately 700,000. What is the predicted population for 2015?

30. The population of Berlin, Germany was about 3,290,000 in 2011. Its population is declining at an annual rate of about 0.2%. What is the predicted population for 2050?
I Like to Move It
Transformations of Exponential Functions

Problem Set

Complete the table to determine the corresponding points on \( c(x) \), given reference points on \( f(x) \). Then, graph \( c(x) \) on the same coordinate plane as \( f(x) \) and state the domain, range, and asymptotes of \( c(x) \).

1. \( f(x) = 2^x \)
   \( c(x) = f(x - 1) \)

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>Corresponding Points on ( c(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{2}))</td>
<td>((0, \frac{1}{2}))</td>
</tr>
<tr>
<td>((0, 1))</td>
<td>((1, 1))</td>
</tr>
<tr>
<td>((1, 2))</td>
<td>((2, 2))</td>
</tr>
</tbody>
</table>

   Domain: All real numbers
   Range: \( y > 0 \)
   Horizontal asymptote: \( y = 0 \)

2. \( f(x) = 4^x \)
   \( c(x) = f(x) - 2 \)

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>Corresponding Points on ( c(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{4}))</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
</tr>
<tr>
<td>((1, 4))</td>
<td></td>
</tr>
</tbody>
</table>

   Domain:
   Range:
   Horizontal asymptote:
3. \( f(x) = 3^x \)
   
   \( c(x) = f(3x) \)

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>Corresponding Points on ( c(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{3}))</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
</tr>
<tr>
<td>((1, 3))</td>
<td></td>
</tr>
</tbody>
</table>

   Domain: 
   Range: 
   Horizontal asymptote:

4. \( f(x) = 2^x \)
   
   \( c(x) = 4f(x) \)

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>Corresponding Points on ( c(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{2}))</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
</tr>
<tr>
<td>((1, 2))</td>
<td></td>
</tr>
</tbody>
</table>

   Domain: 
   Range: 
   Horizontal asymptote:
5. \( f(x) = 5^x \)
   \( c(x) = f(x + 3) \)

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>Corresponding Points on ( c(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{5}))</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
</tr>
<tr>
<td>((1, 5))</td>
<td></td>
</tr>
</tbody>
</table>

Domain:  
Range:  
Horizontal asymptote: 

6. \( f(x) = 4^x \)
   \( c(x) = f(x) + 2 \)

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>Corresponding Points on ( c(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{4}))</td>
<td></td>
</tr>
<tr>
<td>((0, 1))</td>
<td></td>
</tr>
<tr>
<td>((1, 4))</td>
<td></td>
</tr>
</tbody>
</table>

Domain:  
Range:  
Horizontal asymptote:
Describe the transformations performed on \( f(x) \) to create \( g(x) \). Then, write an equation for \( g(x) \) in terms of \( f(x) \).

7. To create \( g(x) \), the graph of \( f(x) \) is compressed horizontally by a factor of \( \frac{1}{2} \) and vertically translated up 1 unit.

\[
g(x) = f(2x) + 1
\]

8.
9. (Graph not visible in text)

10. (Graph not visible in text)
11. \[ f(x) = \begin{cases} 2 & \text{if } x = 1 \\ 4 & \text{if } x = 0 \\ 2 & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases} \]

12. \[ g(x) = \begin{cases} 2 & \text{if } x = 1 \\ 4 & \text{if } x = 0 \\ 6 & \text{if } x = 2 \\ 10 & \text{otherwise} \end{cases} \]
Describe the transformations performed on \( m(x) \) that produced \( t(x) \). Then, write an exponential equation for \( t(x) \).

13. \( m(x) = 3^x \)
   \[ t(x) = -m(x + 1) \]
   The graph of the function \( m(x) \) is reflected over the \( x \)-axis and horizontally translated left 1 unit to produce \( t(x) \).
   \[ t(x) = -3^{x+1} \]

14. \( m(x) = 5^x \)
   \[ t(x) = 3m(x) - 2 \]

15. \( m(x) = e^x \)
   \[ t(x) = \frac{1}{2}m(x) + 4 \]

16. \( m(x) = 4^x \)
   \[ t(x) = m(3x - 1) \]
17. \( m(x) = 7^x \)
\[ t(x) = m(0.5x + 2) \]

18. \( m(x) = 6^x \)
\[ t(x) = -2m(-x) + 3 \]
I Feel the Earth Move
Logarithmic Functions

Vocabulary
Write the term that best completes each sentence.

<table>
<thead>
<tr>
<th>logarithm</th>
<th>logarithmic function</th>
<th>common logarithm</th>
<th>natural logarithm</th>
</tr>
</thead>
</table>

1. The ___________________________ of a number for a given base is the exponent to which the base must be raised in order to produce that number.

2. A ___________________________ is a logarithm with base e, and is usually written as $\ln$.

3. A ___________________________ is a function involving a logarithm.

4. A ___________________________ is a logarithm with a base 10 and is usually written without a base specified.

Problem Set
Write each exponential equation as a corresponding logarithmic equation.

1. $3^2 = 9$
   $\log_3 (9) = 2$

2. $5^4 = 625$
   
3. $4^{-3} = \frac{1}{64}$
   
4. $10^{-5} = \frac{1}{100,000}$
   
5. $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$
   
6. $\left(\frac{1}{11}\right)^{-2} = 121$
Write each logarithmic equation as a corresponding exponential equation.

7. \( \log_7 \left( \frac{1}{49} \right) = -2 \)
   \[ 7^{-2} = \frac{1}{49} \]

8. \( \log_3 \left( \frac{1}{729} \right) = 6 \)

9. \( \log_2 (128) = 7 \)

10. \( \log_6 \left( \frac{1}{1296} \right) = -4 \)

11. \( \log_\frac{1}{3} \left( \frac{1}{125} \right) = 3 \)

12. \( \log_9 (729) = 3 \)

Graph the inverse of each exponential function \( f(x) \). Then, describe the domain, range, asymptotes, and end behavior of the inverse.

13. \( f(x) = 3^x \)

   Domain: \( x > 0 \)
   Range: All real numbers
   Asymptotes: \( x = 0 \)
   End behavior: \( \text{As } x \to 0, y \to -\infty. \)
   \( \text{As } x \to +\infty, y \to +\infty. \)

14. \( f(x) = 2^x \)

   Domain: \( x \in \mathbb{R} \)
   Range: All positive real numbers
   Asymptotes: \( x = 0 \)
   End behavior: \( \text{As } x \to -\infty, y \to 0. \)
   \( \text{As } x \to +\infty, y \to +\infty. \)
15. $f(x) = 10^x$

- Domain:
- Range:
- Asymptotes:
- End behavior:

16. $f(x) = 4^x$

- Domain:
- Range:
- Asymptotes:
- End behavior:
LESSON 10.4 Skills Practice

17. \( f(x) = e^x \)

18. \( f(x) = 5^x \)

Solve each logarithmic equation.

19. \(-2 = \log_b \left(\frac{1}{b}\right)\)
   \[ \frac{1}{b} = 9^{-2} \]
   \[ \frac{1}{b} = \frac{1}{81} \]
   \[ b = 81 \]

20. \(-0.903 \approx x \cdot \log (0.5)\)

21. \(1/2 = \log_n (3)\)

22. \(2.398 = \log b\)

23. \(0.058 = \ln z\)

24. \(-1.349 = \frac{1}{2} \log \left(\frac{g}{1000}\right)\)
More Than Meets the Eye
Transformations of Logarithmic Functions

Problem Set

Analyze the graphs of \( f(x) \) and \( g(x) \). Describe the transformations performed on the graph of \( f(x) \) to produce the graph of the transformed function \( g(x) \). Then, write an equation for \( g(x) \).

1. The graph of \( g(x) \) was horizontally translated right 4 units to produce the graph of \( g(x) \).

\[ g(x) = \log_2 (x - 4) \]
3. $f(x) = \log_4(x)$

4. $f(x) = \log_5(x)$

5. $f(x) = \log_4(x)$

6. $f(x) = \log_2(x)$
The graph of \( f(x) = \log(x) \) is shown. Use the graph of \( f(x) \) to sketch the transformed function \( m(x) \) on the coordinate plane. Then, state the domain, range and asymptotes of \( m(x) \).

7. \( m(x) = f(x) + 2 \).

**Domain of** \( m(x) \): \((0, \infty)\)

**Range of** \( m(x) \): \((-\infty, \infty)\)

**Asymptote of** \( m(x) \): \( x = 0 \)

8. \( m(x) = f(x - 4) \).

**Domain of** \( m(x) \): 

**Range of** \( m(x) \): 

**Asymptote of** \( m(x) \): 
9. \( m(x) = f(x + 1) - 3 \).

- **Domain of** \( m(x) \):
- **Range of** \( m(x) \):
- **Asymptote of** \( m(x) \):

10. \( m(x) = -f(2x - 3) \).

- **Domain of** \( m(x) \):
- **Range of** \( m(x) \):
- **Asymptote of** \( m(x) \):
11. \( m(x) = 0.5f(-x) + 1 \)

Domain of \( m(x) \): 

Range of \( m(x) \): 

Asymptote of \( m(x) \):

12. \( m(x) = -f(x - 2) + 2 \)

Domain of \( m(x) \): 

Range of \( m(x) \): 

Asymptote of \( m(x) \):
Write a transformed logarithmic function, $c(x)$, in terms of $f(x) = \log_2(x)$ with the characteristics given.

13. vertical asymptote at $x = 6$
   Answers will vary.
   
   $c(x) = f(x - 6)$

15. Reference Points on $f(x)$ → Corresponding Points on $c(x)$

<table>
<thead>
<tr>
<th>Reference Points on $f(x)$</th>
<th>→</th>
<th>Corresponding Points on $c(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left( \frac{1}{2}, -1 \right)$</td>
<td>→</td>
<td>$\left( \frac{1}{2}, -3 \right)$</td>
</tr>
<tr>
<td>(1, 0)</td>
<td>→</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>→</td>
<td>(2, 3)</td>
</tr>
</tbody>
</table>

16. vertical asymptote at $x = -2$

17. domain: $(4, \infty)$
    point: $(5, -4)$

18. Reference Points on $f(x)$ → Corresponding Points on $c(x)$

<table>
<thead>
<tr>
<th>Reference Points on $f(x)$</th>
<th>→</th>
<th>Corresponding Points on $c(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left( \frac{1}{2}, -1 \right)$</td>
<td>→</td>
<td>$\left( \frac{1}{2}, 3 \right)$</td>
</tr>
<tr>
<td>(1, 0)</td>
<td>→</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>→</td>
<td>(2, 1)</td>
</tr>
</tbody>
</table>
Consider the function \( y = f(x) \) and the transformed function \( g(x) \). Write an equation for \( g^{-1}(x) \) in terms of \( f^{-1}(x) \).

19. \( g(x) = f(x) + 3 \)
   \[ g^{-1}(x) = f^{-1}(x - 3) \]

20. \( g(x) = f(x - 2) \)

21. \( g(x) = f(x + 6) \)

22. \( g(x) = f(x) - 5 \)

23. \( g(x) = f(x + 1) - 3 \)

24. \( g(x) = f(x - 4) + 2 \)

Consider the function \( f(x) = 4^x \) and its inverse function \( f^{-1}(x) = \log_4(x) \). Complete a table for each transformation and write the transformation function in terms of \( f^{-1}(x) \). Then, identify the transformation on \( f(x) \) and the effect on the inverse.

25. 

<table>
<thead>
<tr>
<th>( h(x) = 2f(x) )</th>
<th>( h^{-1}(x) = f^{-1}\left(\frac{x}{2}\right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, \frac{1}{2}))</td>
<td>(\left(\frac{1}{2}, -1\right))</td>
</tr>
<tr>
<td>((0, 2))</td>
<td>((2, 0))</td>
</tr>
<tr>
<td>((1, 8))</td>
<td>((8, 1))</td>
</tr>
</tbody>
</table>

Transformation on \( f(x) \): vertical dilation of 2

Effect on the inverse: horizontal dilation of 2
### Lesson 10.5 Skills Practice

**Page 8**

#### 26.

<table>
<thead>
<tr>
<th>$m(x) = \frac{1}{4}f(x)$</th>
<th>$m^{-1}(x)$ =</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

**Transformation on $f(x)$:**

**Effect on the inverse:**

#### 27.

<table>
<thead>
<tr>
<th>$h(x) = f(8x)$</th>
<th>$h^{-1}(x)$ =</th>
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<tbody>
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</table>

**Transformation on $f(x)$:**

**Effect on the inverse:**

#### 28.

<table>
<thead>
<tr>
<th>$h(x) = f\left(\frac{x}{2}\right)$</th>
<th>$h^{-1}(x)$ =</th>
</tr>
</thead>
<tbody>
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**Transformation on $f(x)$:**

**Effect on the inverse:**
Name ___________________________ Date ____________

29. \( h(x) = 12f(x) \) \( h^{-1}(x) = \)

<table>
<thead>
<tr>
<th>Transformation on ( f(x) ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect on the inverse:</td>
</tr>
</tbody>
</table>

30. \( h(x) = f(12x) \) \( h^{-1}(x) = \)

<table>
<thead>
<tr>
<th>Transformation on ( f(x) ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect on the inverse:</td>
</tr>
</tbody>
</table>
Given each function \( f(x) \), write an equation for the inverse function \( f^{-1}(x) \).

31. \( f(x) = 2^x \)  
   \( f^{-1}(x) = \frac{1}{3} \log_2(x) \)

32. \( f(x) = 4 \cdot 3^x \)  
   \( f^{-1}(x) = \)

33. \( f(x) = \log_3 \left( \frac{x}{3} \right) \)  
   \( f^{-1}(x) = \)

34. \( f(x) = \frac{1}{4} \log_4(x) \)  
   \( f^{-1}(x) = \)

35. \( f(x) = \log(-x) \)  
   \( f^{-1}(x) = \)

36. \( f(x) = 2 \cdot 4^x \)  
   \( f^{-1}(x) = \)